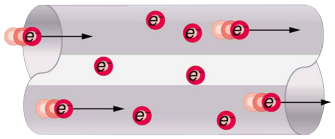
Free Charge in a Copper Wire

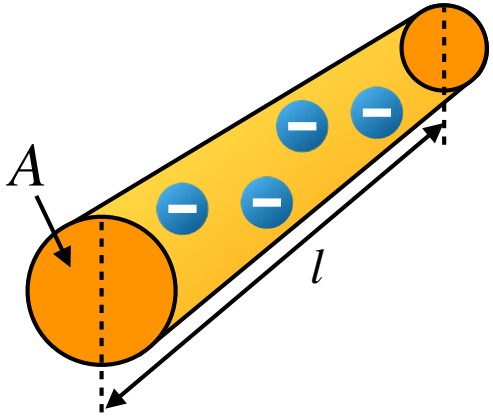
Solution method



Step 1 : Find out the knowns and unknowns of the task.

What we know

- Cross-sectional area $A = 2.5\ mm^2$
- Wire length $l = 1\ m$
- Atomic weight of copper $a_{Cu} = 63.546\ g/mol$
- Avogadro's number : $6.022 \cdot 10^{23}\ atoms/mol$
- Density of copper $\rho_{Cu} = 8.96\ g/cm^3$
- Elementary charge $e^- = 1.602 \cdot 10^{-19}\ C$



What we <u>want</u> to know

- Amount of free charge in the copper wire

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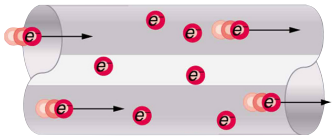


Step 2: Find out the number of copper atoms in $1m^3$

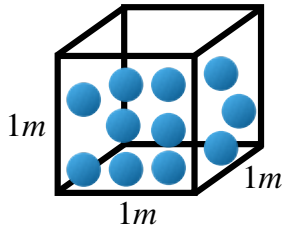
We can calculate the number of atoms in $1m^3$ of copper using ***Avogadro*'s constant**

$$n_{a,m^3} = \frac{\rho_{Cu}}{a_{Cu}} \cdot 10^6 = 141 \cdot 10^3 \; mol/m^3$$

$$(1mol \equiv 6.022 \cdot 10^{23} \; atoms)$$

For copper, the **number of atoms** results to

$$n_{a,m^3} = 8.491 \cdot 10^{28} atoms/m^3$$



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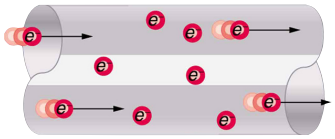
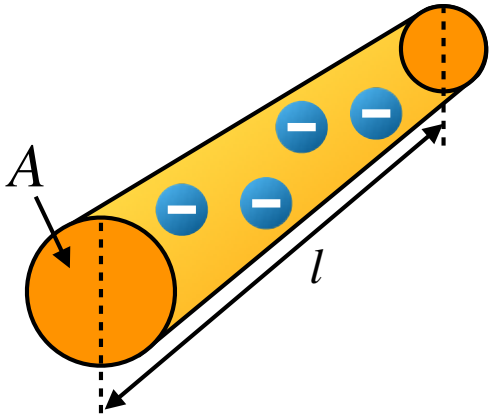
Solution method



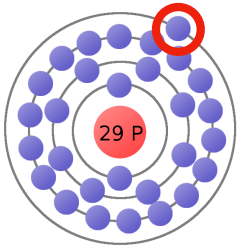
Step 3: Calculate the amount of free charge in the wire

As copper atoms in a lattice have **one valence electron** each, $n_{a,wire}$ is also the **number of free elementary charges** in the wire.



$$V_{wire} = A \cdot l$$

$$n_{a,wire} = n_{e^-,wire} = A \cdot l \cdot n_{a,m^3}$$



$$\boxed{Q = e^- \cdot A \cdot l \cdot n_{a,m^3}}$$

$$Q = 1.602 \cdot 10^{-19}\ C \cdot 2.5 \cdot 10^{-6}\ m^2 \cdot 1\ m \cdot 8.49 \cdot 10^{28}\ atoms/m^3 = 34 \cdot 10^3\ C$$

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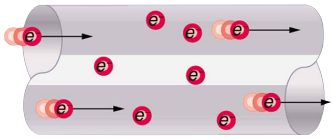
Solution method



Step 3: Calculate the amount of free charge in the wire

Aluminum vs. Copper

$$Q_{cu} = 34 \cdot 10^3 C$$

$$n_{a,al} = 1.808 \cdot 10^{29}\ electrons/m^3$$

$$Q_{al} = 72.4 \cdot 10^3 C$$

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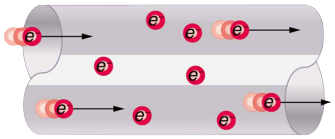
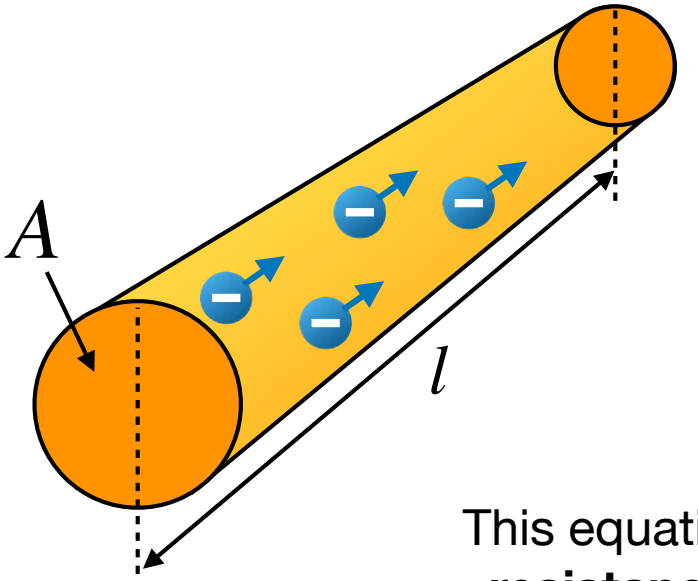


Step 4: Find the charge passing through the wire in a given time.



$$\frac{Q}{t} = \frac{e^- \cdot A \cdot l \cdot n_{a,m^3}}{t}$$

This equation will be the basis for defining **current**, **voltage** and **resistance** as well as for deriving **Ohm's law** (see next video)